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# The fundamental plane of elliptical galaxies with modified Newtonian dynamics

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## ABSTRACT

The modified Newtonian dynamics (MOND), suggested by Milgrom as an alternative to dark matter, implies that isothermal spheres with a fixed anisotropy parameter should exhibit a near perfect relation between the mass and velocity dispersion of the form  $M \propto \sigma^4$ . This is consistent with the observed Faber-Jackson relation for elliptical galaxies– a luminosity-velocity dispersion relation with large scatter. However, the observable global properties of elliptical galaxies comprise a three parameter family; they lie on a “fundamental plane” in a logarithmic space consisting of central velocity dispersion, effective radius ( $r_e$ ), and luminosity. The scatter perpendicular to this plane is significantly less than that about the Faber-Jackson relation. I show here that, in order to match the observed properties of elliptical galaxies with MOND, models must deviate from being strictly isothermal and isotropic; such objects can be approximated by high-order polytropic spheres with a radial orbit anisotropy in the outer regions. MOND imposes boundary conditions on the inner Newtonian regions which restrict these models to a dynamical fundamental plane of the form  $M \propto \sigma^\alpha r_e^\gamma$  where the exponents may differ from the Newtonian expectations ( $\alpha = 2$ ,  $\gamma = 1$ ). Scatter about this plane is relatively insensitive to the necessary deviations from homology.

**Key words:** galaxies: elliptical– galaxies: kinematics and dynamics– dark matter, gravitation

## 1 INTRODUCTION

On a phenomenological level, the most successful alternative to cosmic dark matter is the modified Newtonian dynamics (MOND) proposed by Milgrom (1983a). The basic idea is that the deviation from Newtonian gravity or dynamics occurs below a fixed acceleration scale– a proposal which is supported in a general way by the fact that discrepancy between the classical dynamical mass and the observable mass in astronomical systems does seem to appear at accelerations below  $10^{-8}$  cm/s<sup>2</sup> (Sanders 1990, McGaugh 1998). The fact that this acceleration scale is comparable to the present value of the Hubble parameter multiplied by the speed of light ( $cH_0$ ) suggests a cosmological basis for this phenomenology.

MOND, in a sense, is designed to reproduce flat extended rotation curves of spiral galaxies and a luminosity-rotation velocity relationship of the observed form,  $L \propto v^4$  (the Tully-Fisher relation). But apart from these aspects which are “built-in”, the prescription also successfully predicts the observed form of galaxy rotation curves from the observed distribution of stars and gas with reasonable values for the mass-to-light ratio of the stellar component (Bege-

man et al. 1991, Sanders 1996, Sanders & Verheijen 1998). Moreover, MOND predicts that the discrepancy between the Newtonian dynamical mass and the observed mass should be large in low surface brightness galaxies– a prediction subsequently borne out by observations of these systems (McGaugh & de Block, 1997, 1998).

The observational success of MOND is most dramatically evident for spiral galaxies where often the rotation curve can be a rather precise tracer of the radial distribution of the effective gravitational force; for hot stellar systems– elliptical galaxies– the predictions are less precise. Milgrom, in his original papers, (1983b) pointed out that MOND implies a mass-velocity dispersion relation for elliptical galaxies of the form  $M \propto \sigma^4$ . If there were no systematic variation of M/L with mass, this, would become the observed Faber-Jackson relation (Faber & Jackson 1976).

In his seminal paper on this subject, Milgrom (1984) calculated the structure of isothermal spheres in the context of MOND and drew several very general conclusions: First, all isothermal spheres, regardless of the degree of anisotropy in the velocity distribution, have finite mass. For a one-dimensional velocity dispersion of 100 to 200 km/s, this mass is inevitably on a galaxy scale. Second, at large radial dis-

tance the density decreases as  $r^{-\delta}$  where  $\delta$  is in the vicinity of 4. Third, there is an absolute maximum on the mean surface density which is on the order of  $a_o/G$  where  $a_o$  is the MOND critical acceleration. For a mass-to-light ratio of three to four in solar units, this would translate into a surface brightness in the V band of 20 to 20.5 mag/(arcsec)<sup>2</sup> which is characteristic of hot stellar systems ranging from massive ellipticals to bulges of spiral galaxies to globular clusters (Corollo et al. 1997). Finally, the mass of an isothermal sphere with a specific anisotropy factor is exactly proportional to  $\sigma^4$  where  $\sigma$  is the space velocity dispersion. All of these conclusions would apply to elliptical galaxies, in so far as these objects can be regarded as isothermal spheres.

Since this work, it has come to light that the global properties of elliptical galaxies comprise a three parameter family (Dressler et al. 1987, Djorovski & Davis 1987); that is to say, elliptical galaxies lie on a surface in a three-dimensional space defined by the luminosity ( $L$ ), the central velocity dispersion ( $\sigma_o$ ), and the effective radius ( $r_e$ ); the mean surface brightness ( $I_e$ ) may be substituted for either luminosity or effective radius (i.e.,  $L = 2\pi I_e r_e^2$ ). This surface appears as a plane on logarithmic plots and, consequently, has been designated as “the fundamental plane” of elliptical galaxies, with the form  $L \propto \sigma^a r_e^b$  where  $a \approx 1.5$  and  $b \approx 0.8$ . Because of the small scatter perpendicular to the fundamental plane, this three-parameter relationship supersedes the Faber-Jackson relation as a distance indicator. The usual physical interpretation of the fundamental plane is that these relations result from the traditional virial theorem plus a dependence of mass-to-light ratio on galaxy mass (van Albada et al. 1995); although, one must also assume that elliptical galaxies comprise a near-homologous family.

It is not immediately clear how the fundamental plane can be interpreted in terms of modified dynamics, or, indeed, if the fundamental plane is even consistent with modified dynamics. The effective virial theorem for a system deep in the regime of MOND (low internal accelerations) is of the form  $\sigma^4 \propto M$ , with no length scale appearing. On the face of it, this would imply that hot stellar systems should comprise a two parameter family as suggested by the older Faber-Jackson relation.

The purpose of this paper is to consider the dynamics of elliptical galaxies in terms of MOND, particularly with respect to the relations between global properties. I demonstrate that the MOND isothermal sphere is actually not a good representation of high surface brightness elliptical galaxies because the implied average surface densities are too low. Elliptical galaxies, within the half-light radius (the effective radius), are essentially Newtonian systems with accelerations in excess of the critical MOND acceleration. This suggests that other possible degrees of freedom must be exploited to model elliptical galaxies using modified dynamics.

To reproduce the observed properties of high surface-brightness elliptical galaxies, it is necessary to introduce small deviations from a strictly isothermal and isotropic velocity field in the outer regions. A simple and approximate way of doing this is to consider high-order polytropic spheres (all of which are finite in the context of MOND) with a velocity distribution which varies from isotropic within a critical radius to highly radial motion in the outer regions. The structure of such objects is determined here by numerical solutions of the hydrostatic equation of stellar dynamics

(the Jeans equation) modified through the introduction of the MOND formula for the gravitational acceleration. I find that all models characterized by a given value of the polytropic index and the appropriately scaled anisotropy radius are homologous and exhibit a perfect mass-velocity dispersion relation of the form  $M \propto \sigma^4$  with no intrinsic scatter. However a range of models over this parameter space is required to reproduce the dispersion in the observed global properties of elliptical galaxies, and strict homology is broken. This adds considerable scatter to the mass-velocity dispersion relation and introduces a third parameter which is, in effect, the surface density (or effective radius). Although these objects are effectively Newtonian in the inner regions, MOND imposes boundary conditions which restrict these Newtonian solutions to a well-defined domain in the three dimensional space of dynamical parameters— a dynamical fundamental plane. Combined with a weak dependence of  $M/L$  on galaxy mass, the fundamental plane in its observed form is reproduced.

Not only the form but also the scaling of the  $M - \sigma_o - r_e$  relation is fixed over the relevant domain of parameter space; this scaling is relatively independent of the detailed structure of the stellar system. Therefore, given the effective radius and velocity dispersion, the mass of any elliptical galaxy may be calculated and the mass-to-light ratio can be determined. For the galaxies in the samples of Jørgensen, Franx, & Kærgaard (1995a,b) and Jørgensen (1999) the mean  $M/L$  turns out to be  $3.6 M_\odot/L_\odot$  with about 30% scatter. With a weak dependence of  $M/L$  with galaxy mass, the predicted form of the Fundamental Plane agrees with that found by Jørgensen et al.

## 2 BASIC EQUATIONS AND ASSUMPTIONS

Following Milgrom (1984), I calculate the structure of spherical systems by integrating the spherically symmetric hydrostatic equation (the Jeans equation):

$$\frac{d}{dr}(\rho\sigma_r^2) + \frac{2\rho\sigma_r^2\beta}{r} = -\rho g \quad (1)$$

where  $\rho$  is the density,  $\sigma_r$  is the radial component of the velocity dispersion,  $\beta = 1 - \sigma_t^2/\sigma_r^2$  is the anisotropy parameter ( $\sigma_t$  is the velocity dispersion in the tangential direction), and  $g$  is the radial gravitational force which, in the context of MOND, is given by

$$g\mu(g/a_o) = \frac{GM_r}{r^2} = \frac{4\pi G}{r^2} \int_0^r r'^2 \rho(r') dr'. \quad (2)$$

Here  $M_r$  is the mass within radius  $r$ ,  $a_o$  is the MOND acceleration parameter (found to be  $1.2 \times 10^{-8}$  cm/s<sup>2</sup> from the rotation curves of nearby galaxies), and  $\mu(x) = x/\sqrt{1+x^2}$  is the typically assumed function interpolating between the Newtonian regime ( $x \gg 1$ ) and the MOND regime ( $x \ll 1$ ). Because of the assumed spherical symmetry, eq. 2 is exact in the context of the Lagrangian formulation of MOND as a modification of Newtonian gravity (Bekenstein & Milgrom 1984). However, if viewed as a modification of inertia eq. 2 may only be an approximation in the general case of motion on non-circular orbits (Milgrom 1994).

There are four unknown functions of radius,  $\rho$ ,  $\sigma_r$ ,  $\beta$ ,

and  $g$ ; therefore, additional assumptions are required to close this set of 2 equations. First, I take a definite pressure-density relation, that of a polytropic equation of state:

$$\sigma_r^2 = A_{n\sigma} \rho^{\frac{1}{n}} \quad (3)$$

where  $A_{n\sigma}$ , for a particular model, is a constant which is specified by the given central velocity dispersion and density, and  $n$  is the polytropic index (for an isothermal sphere  $n$  is infinite). This is done simply as a convenient way of providing stellar systems which are somewhat cooler in the outer regions as is suggested by observations (discussed below). The anisotropy parameter is further assumed to depend upon radius as

$$\beta(r) = \frac{(r/r_a)^2}{[1 + (r/r_a)^2]} \quad (4)$$

where  $r_a$  is the assumed anisotropy radius. This provides a velocity distribution which is isotropic within  $r_a$ , but which approaches pure radial motion when  $r \gg r_a$ . Such behavior is typical of systems which form by dissipationless collapse (van Albada 1982). Thus I will be considering a two-parameter set of models for elliptical galaxies characterized by a polytropic index  $n$  and an anisotropy radius  $r_a$ . Additional physical considerations, such as stability, may limit the range of these parameters.

Milgrom (1984) found that for an isothermal sphere with a given constant  $\beta$ , there exists a family of MOND solutions having different asymptotic behavior as  $r \rightarrow 0$ . In general, the density approaches a constant value near the center except for one particular limiting solution where  $\rho \rightarrow 1/r^2$ . The global properties (e.g., the mean surface density, the value of  $M/\sigma^4$ , the asymptotic density distribution at large  $r$ ) do not vary greatly within such a family of solutions, except for models which are characterized by an unrealistically low central density of  $< 10^{-2} M_\odot/\text{pc}^3$  (see Milgrom 1984, Figs. 1 & 2). I find the same to be true for the high-order polytropic spheres with an anisotropy factor given by eq. 4; i.e., for a given value of  $n$  and  $r_a$ , there is a family of solutions with differing asymptotic behavior at small  $r$ . Again, because the global parameters do not vary greatly within a family, I consider below only the limiting solution; i.e., that with the  $1/r^2$  density cusp.

Before describing the results of the numerical integration of eqs. 1-4, one important aspect of this system of equations should be emphasized. The appearance of an additional dimensional constant,  $a_o$ , in the equation for the gravitational force, eq. 2, provides, when combined with a characteristic velocity dispersion of the system (the central radial velocity dispersion, for example), a natural length and mass scale for objects described by these equations. This differs from the case of Newtonian systems where two system parameters (a velocity dispersion and central density) are required to define units of mass and density. For MOND systems the characteristic length and mass are given by

$$R_\sigma = \sigma_r^2 / a_o, \quad (5a)$$

$$M_\sigma = \sigma_r^4 / G a_o. \quad (5b)$$

There is, in addition, a natural scale for surface density given by

$$\Sigma_m = M_\sigma / R_\sigma^2 = a_o / G \quad (5c)$$

which depends only upon fundamental constants. These natural units imply that the properties of homologous systems described by modified dynamics should scale according to these relations; i.e.,  $M \propto \sigma^4$  with a characteristic surface density which is independent of  $\sigma$ . However, for the system to be described by modified dynamics it must extend into the regime of low accelerations, i.e., where  $a \ll a_o$ . This is not a necessary attribute of Newtonian systems which have finite mass and radius ( $n < 5$ ). Therefore, we would expect that only those higher order polytropes ( $n > 5$  including the isothermal sphere) to be necessarily characterized by this scaling because the Newtonian solution always has infinite extent and mass. Such polytropes would inevitably extend to the regime of low accelerations.

For a velocity dispersion of 200 km/s, typical for elliptical galaxies, we find  $R_\sigma \approx 10$  kpc and  $M_\sigma \approx 10^{11} M_\odot$ —characteristic galactic dimensions. The manner in which these length and mass scales arise, in the context of a cosmological setting for the dissipationless formation of elliptical galaxies, will be discussed in future paper.

### 3 A TEST CASE: THE ISOTROPIC ISOTHERMAL SPHERE

Eq. 1 is numerically solved using a fourth-order Runge-Kutte technique. The integration proceeds radially outward by specifying a central radial velocity dispersion and choosing, at a particular radius, the density corresponding to that of the limiting solution given by Milgrom (1984). In solving for the structure of isothermal spheres, Milgrom used the natural units of length and mass (eq. 5) to write eq. 1 in unitless form. Here, I use physical units: 1 kpc,  $10^{11} M_\odot$ , and 1 km/s (in these units  $G = 4.32 \times 10^5 (\text{km/s})^2 \text{kpc} / (10^{11} M_\odot)$  and  $a_o = 3700 (\text{km/s})^2 / \text{kpc}$ ). Although convenient for comparison with observations, these units do obscure the scaling of solutions.

For comparison with Milgrom's results the first systems considered here are isotropic isothermal spheres ( $n, r_a \rightarrow \infty$ ). The limiting solution of an isothermal sphere with a specified value of  $\beta$  can be scaled as implied by the natural units (eqs. 5). I verify this by solving eqs. 1-4 for isothermal spheres with radial velocity dispersion ranging from 50 km/s to 350 km/s. For all such spheres, the density at large radius falls off as  $1/r^4$ . This implies, directly from eq. 1, that

$$\sigma_r^4 = 0.063 G M a_o \quad (6)$$

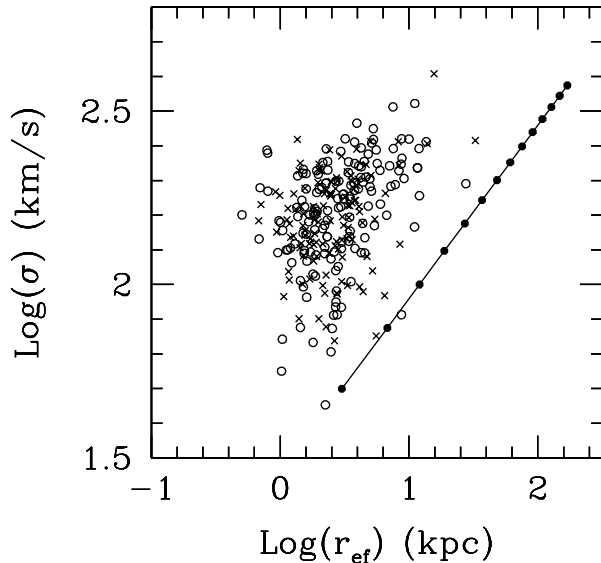
which is to say, the mass-velocity dispersion relation is exact for MOND isotropic isothermal spheres.

Because the density distribution for the limiting solution falls as  $1/r^2$  at small  $r$  and as  $1/r^4$  in the outer regions, the density profile of the MOND isotropic isothermal sphere resembles that of the Jaffe model (Jaffe 1983). The mean surface density inside the projected half-mass radius (the effective radius),  $r_e$ , is found to be

$$\Sigma = 0.134 \Sigma_m. \quad (7)$$

where the characteristic MOND surface density,  $\Sigma_m$ , is given by eq. 5c. Given that  $M = 2\pi r_e^2 \Sigma$ , then from eqs. 6 and 7 it follows that

$$r_e = 4.36 \sigma_r^2 / a_o. \quad (8)$$



**Figure 1.** A log-log plot of the central velocity dispersion (km/s) vs. the effective radius (kpc) for cluster elliptical galaxies (open points, Jørgensen et al. 1995), and for early type galaxies in the Coma cluster (crosses, Jørgensen 1999). The connected solid points show the sequence of MOND isotropic isothermal spheres.

For a system with  $\sigma_r = 125$  km/s, one would find, with these formulae, that  $M = 2.4 \times 10^{11} M_\odot$  and  $r_e = 18$  kpc. The surface density distribution of such an object is well-described by the de Vaucouleurs  $r^{1/4}$  law which works well as an empirical fit to surface brightness profile of elliptical galaxies.

However, the global properties of MOND isotropic isothermal spheres are inconsistent with those of high surface-brightness elliptical galaxies. Basically, for a given velocity dispersion, the mass is too large, the effective radius is too large, and the mean surface density is too small compared to that of actual galaxies. This is evident from Fig. 1 which is a logarithmic plot of the central velocity dispersion plotted against effective radius for a large homogeneously observed sample of galaxies. The sample consists of 154 elliptical galaxies in several clusters (open points) observed by Jørgensen et al. (1995a,b) combined with 116 early-type galaxies (crosses) in the Coma cluster (Jørgensen 1999). The connected solid points show the sequence of isotropic isothermal spheres (eq. 8). We see that the distribution of observed properties is not matched with those of the isothermal spheres; for example, for galaxies with a central line-of-sight (l.o.s.) velocity dispersion of about 150 km/s, the effective radii are typically 2.5 kpc, an order of magnitude lower than that of the MOND isothermal spheres. Assuming a mass-to-light ratio of 3 to 4  $M_\odot/L_\odot$  for the stellar population, the mean observed surface density in actual elliptical galaxies within  $r_e$  would be about two or three times larger than  $\Sigma_m$ ; which is to say, elliptical galaxies are well within the Newtonian regime inside an effective radius. In contrast the mean surface density of MOND isotropic isothermal spheres are at least 10 times lower (eq. 7) and hence fail as representations of luminous high surface brightness elliptical galaxies.

The situation becomes worse for constant  $\beta > 0$  as is evident from the calculation of Milgrom (1984). Such systems are even deeper in the MOND regime; for  $\beta = 0.9$ ,  $\Sigma = 0.0025 \Sigma_m$ . In order to represent elliptical galaxies in the context of MOND, we must allow for the possibility that pressure-supported systems become cooler with a radial orbit anisotropy in the outer regions.

#### 4 ANISOTROPIC POLYTROPES AS MODELS FOR ELLIPTICAL GALAXIES

There is observational evidence that the stellar component of elliptical galaxies is not, in general, isothermal: the l.o.s. velocity dispersion is observed to decrease with increasing projected radius. For reasons of stability such a decrease can probably not be attributed entirely to velocity field anisotropy (see below). Describing this decline by a power law,  $\sigma \propto r^{-\epsilon}$ , Franx (1989) finds that typically  $\epsilon = 0.06$ . A simple way of introducing this mild deviation from an isothermal state into the MOND models is to consider the more general equation of state expressed by eq. 3– the polytropic gas assumption with large  $n$ .

It is straight-forward to demonstrate from eq. 1 that all MOND polytropic spheres of finite  $n$  are finite in extent as well as in mass, unlike the Newtonian case where only polytropes with  $n < 5$  have finite extent and mass. As  $n \rightarrow \infty$ , the radius of the edge of the sphere also approaches infinity; for high order polytropes ( $n > 10$ ) the outer radius is many times larger than the effective radius. A polytropic sphere with  $n > 5$  will always have a MOND regime which establishes boundary conditions for the inner Newtonian solution. Therefore, the polytropic index, which must lie between 5 and infinity, is a free parameter of such models.

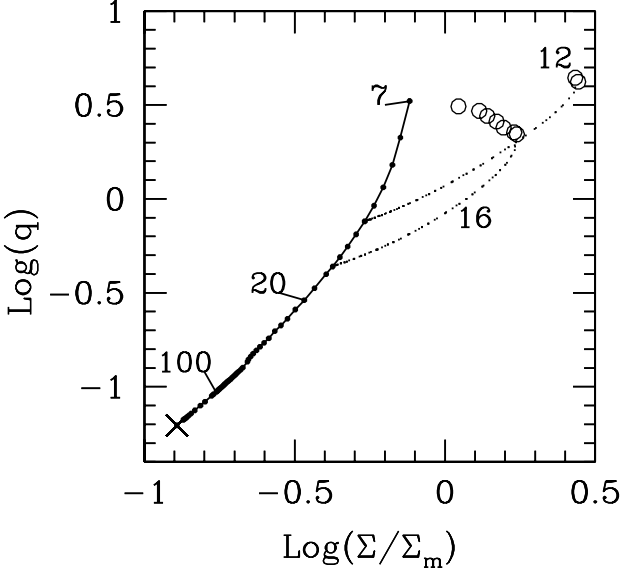
It is unlikely that the stars in elliptical galaxies have a completely isotropic velocity distribution. As noted above, a radial orbit anisotropy of the form given by eq. 4 emerges naturally in dissipationless collapse models. This expression provides a second dimensionless parameter for characterizing MOND models of elliptical galaxies:  $\eta = r_a/R_\sigma$ , the anisotropy radius in terms of the characteristic MOND length scale

A very general result is that such high order anisotropic polytropic spheres have a velocity dispersion-mass relation of the form,

$$\sigma_o^4 = q(n, \eta) G M a_o \quad (9)$$

where  $\sigma_o$  is the *central* l.o.s. velocity dispersion. That is to say, the ratio of  $\sigma_o^4$  to  $G M a_o$ , defined here as  $q$ , depends upon the two dimensionless parameters  $n$  and  $\eta$ . This follows from the general scaling relation for systems which extend into the regime of modified dynamics, eq. 5b. For the pure isotropic isothermal sphere  $q_\infty = 0.063$  (eq. 6). For any given polytropic index and scaled anisotropy radius, the structure of objects is self-similar over central radial velocity dispersion, and the  $M - \sigma_o^4$  relation is exact; i.e., such objects form a 2-parameter family in  $\sigma_o$  and  $M$ .

The same is not true if we consider the set of such systems over a domain of the two-dimensional parameter space. In Fig. 2 the connected solid points show the logarithm of  $q$  vs. the logarithm of the mean surface density within the effective radius ( $\Sigma = M/2\pi r_e^2$ ) in units of the MOND critical



**Figure 2.** A log-log plot of  $q$  which is the ratio  $\sigma_o^4/GMa_o$  ( $\sigma_o$  is the central l.o.s. velocity dispersion and  $M$  is the galaxy mass) against the mean surface density within  $r_e$  in terms of the MOND surface density ( $a_o/G$ ) for MOND polytropic spheres. The principal branch shown by the connected solid points is a sequence of isotropic polytropes; each point is a polytropic sphere of a given index  $n$ . The ‘X’ shows the position of the isothermal sphere and spheres with  $n=100$ , 20, and 7 are also indicated. The parameter  $q$  is the scale factor in the mass-velocity dispersion relation, so each polytropic sphere has its own  $M - \sigma_o^4$  relation. Branching off of this curve, for polytropes of  $n=16$  and  $n=12$  are sequences of anisotropic models where the anisotropy radius, in terms of the MOND length scale ( $\sigma_o^2/a_o$ ) decreases from 200 down to 0.1. It is seen that increasing anisotropy moves models to regions of higher surface density. The open points indicate highly anisotropic models with  $r_a < 0.75r_e$ .

surface density  $\Sigma_m$  for a sequence of isotropic polytropes ( $\eta = \infty$ ). Each point is a set of models over a range of central  $\sigma_o$  but having a specific value of  $n$ . The locus of such points define a curve with the isothermal sphere (denoted by an X) at one extreme and, as displayed here, the  $n=7$  polytrope at the other. The sequence of isotropic polytropes approaches but does not exceed the MOND critical surface density. Each polytrope is self-similar over velocity dispersion with a perfect mass-velocity dispersion relation, but its own mass-velocity relation. The entire set of polytropes breaks homology, and for this ensemble of isotropic polytropes,  $q$  can be considered as a function of the mean surface density; indeed, over the range of  $n=30$  to  $n \rightarrow \infty$ , the curve is well approximated by a power law, i.e.,

$$q = k(\Sigma/\Sigma_m)^\kappa \quad (10)$$

That is to say, a third parameter, the mean surface density, enters into the basic dynamical relationship, eq. 9. Substituting eq. 10 into eq. 9 we find a relation of the form

$$\sigma_o^4 = k \frac{G^{\kappa+1} M^{\kappa+1}}{2\pi r_e^{2\kappa} a_o^{\kappa-1}} \quad (11)$$

This can be viewed as a generalized virial relationship for objects which extend into the regime of modified dynamics.

For models with a specified value of  $n$  and  $\eta$ ,  $\kappa = 0$  and we recover the MOND mass-velocity dispersion relationship for homologous systems; models covering a range of  $n$  and  $\eta$ – non-homologous models with  $\kappa \neq 0$ – comprise a three-parameter family.

If structure of elliptical galaxies could be approximated by this set of high order isotropic polytropic spheres ( $30 < n < \infty$ ), then, from eq. 11, there would exist a theoretical fundamental plane relationship of the form

$$M = K \sigma_o^\alpha r_e^\gamma \quad (12a)$$

where

$$\alpha = \frac{4}{\kappa + 1}, \quad (12b)$$

$$\gamma = \frac{2\kappa}{\kappa + 1} \quad (12c)$$

and

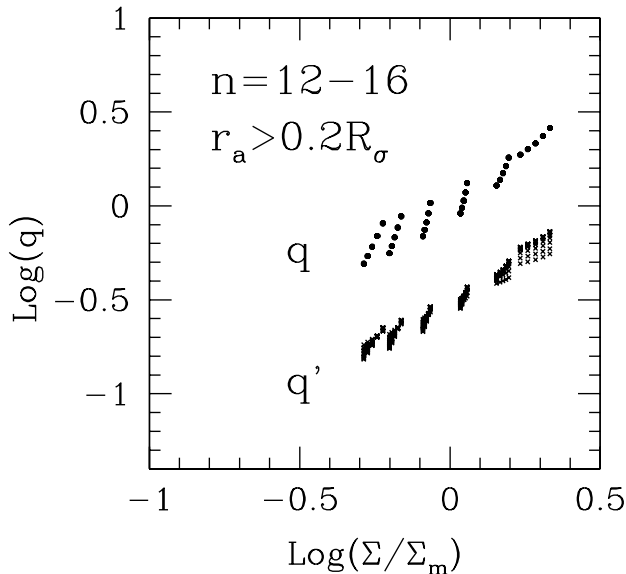
$$K = \left( \frac{2\pi}{k G^{\kappa+1} a_o^{1-\kappa}} \right)^{\frac{1}{\kappa+1}}. \quad (12d)$$

For the sequence of isothermal spheres over this range of  $n$ ,  $\kappa = 1.5$  which, from eqs. 12b and 12c, implies that  $\alpha = 1.6$  and  $\gamma = 1.2$ . Thus, in this generalized dynamical relation, the exponents may differ from the expected Newtonian values ( $\alpha = 2$ ,  $\gamma = 1$ ). *Significantly, this one dynamical formula (eq. 12a) applies to a range of models which are non-homologous.*

However, pure isotropic polytropic spheres also fail as acceptable models of elliptical galaxies. In the models, as in actual ellipticals, the l.o.s. velocity dispersion declines with increasing projected radius. This decline, mild though it is, is steeper than that typically observed in ellipticals if  $n < 12$  and too shallow if  $n > 16$ . MOND polytropes in the range of  $n=12$  to  $n=16$  exhibit roughly the observed form of  $\sigma(r)$ . For polytropes in this range, the mean surface density within the effective radius is still significantly lower than that of true elliptical galaxies– typically by about a factor of 5 assuming a mass-to-light ratio of 4 for stellar population of elliptical galaxies. If such simple spherically symmetric models are to approximate real elliptical galaxies, it is clear an additional degree of freedom must enter into the structure equation (eq. 1). Here we assume that that degree of freedom is provided by the radial dependence of the anisotropy parameter as represented by eq. 4.

In Fig. 2 we see the effect of introducing this second parameter,  $\eta$ , on the position of polytropes in the  $\log(q)$ - $\log(\Sigma)$  plane. Branching off of the principal curve defined by the sequence of isotropic polytropes are sequences of models with  $\eta$  ranging from 200 to 0.1 for polytropes of  $n=12$  and  $n=16$ . When  $\eta \gg 1$  the models, of course, are very similar to the MOND isotropic polytropes. The effect of increasing anisotropy (lower  $\eta$ ) is to increase the mean surface density of the polytropic spheres– as is needed to match the observations of elliptical galaxies. The mass and effective radius are decreased but the mean surface density is higher.

For a given polytropic index, the branch defined by increasing anisotropy (decreasing  $\eta$ ) exhibits a maximum surface density; for  $n=16$  this maximum is about  $2\Sigma_m$  and occurs for  $\eta \approx 0.15$ ; that is, for lower  $\eta$  (higher anisotropy) the surface brightness is again lower. The sequence of anisotropic

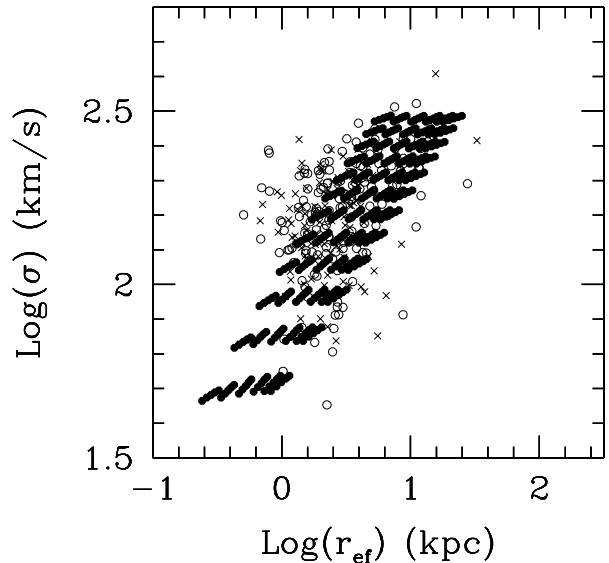


**Figure 3.** The  $\log(q)$ - $\log(\Sigma)$  plot for polytropes between  $n=12$  and  $n=16$ . The scaled anisotropy radius is greater than 0.2. Each point represents a model with specific values of the polytropic index and scaled anisotropy radius. There is a perfect mass-velocity dispersion relation for each point on this plot; although, the ensemble of points would exhibit a  $M - \sigma_o^4$  relation with considerable scatter since the normalization ( $q$ ) varies by a factor of five. Also shown is  $q'$  which is the normalization of the mass-velocity dispersion relation where the intensity-weighted mean l.o.s. velocity dispersion,  $\sigma_d$ , within a circular diaphragm of radius  $r_d = 0.8$  kpc is substituted for the central velocity dispersion; i.e.,  $q' = \sigma_d^{4-\lambda} \sigma_m^\lambda / G M a_o$  with  $\sigma_m = \sqrt{a_o r_d}$ . This is the mass-velocity dispersion relation which is relevant to actual observations of ellipticals (see text).

models is double-valued in surface brightness. These models near the maximum surface density are quite anisotropic in the sense that the radial orbit anisotropy reaches within the effective radius; all models with  $r_a < 0.75 r_e$  are designated by an open circle in Fig. 2. The stability of such anisotropic models is questionable (Binney & Tremaine 1987).

Given the fact that MOND anisotropic polytropes between  $n=12$  and  $n=16$  can reproduce the approximate decline of the l.o.s. velocity dispersion with projected radius observed in ellipticals, we can take this as an observational restriction upon the range of  $n$ . The range of the second parameter,  $\eta$ , can also be restricted by excluding all highly anisotropic models (with  $\eta < 0.2$ ) on the basis of possible radial orbit instability. Fig. 3 is the locus of a grid of such models on the  $\log(q)$ - $\log(\Sigma)$  plane. There are 360 models with  $n = 12, 13, 14, 15, 16$  and  $\eta = 0.2, 0.4, 0.8, 1.6, 3.2, 6.4$  and covering a range of the central  $\sigma_r$  between 75 km/s and 350 km/s in steps of 25 km/s.

Each point in Fig. 3 represents a particular value of  $n$  and  $\eta$  and exhibits its own perfect  $M - \sigma_o^4$  relation. However, the ensemble of models is non-homologous and presents an ensemble of  $M - \sigma_o^4$  relations. Because  $q$  varies by a factor of 5 or 6 this would be the expected intrinsic scatter in the combined  $M - \sigma_o$  relation. However, over this range of parameter space, the models lie in a restricted domain of

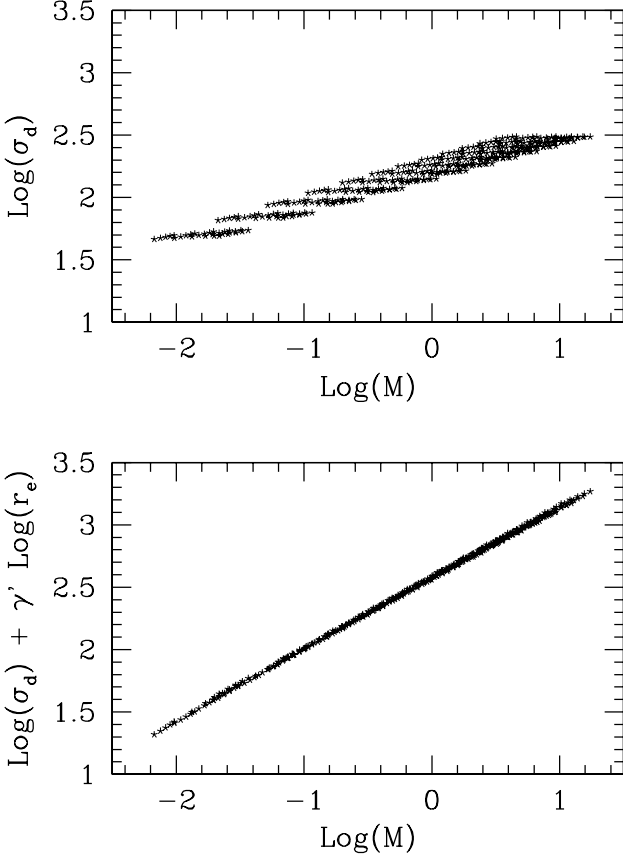


**Figure 4.** The l.o.s. velocity dispersion,  $\sigma_d$ , within  $r_d$  plotted against the effective radius (as in Fig. 1) again for the galaxies from the samples of Jørgensen et al. but here compared to the ensemble of MOND anisotropic polytropes (solid points); i.e., the polytropic index ranges from 12 to 16 with a scaled anisotropy radius greater than 0.2.

the  $\log(\Sigma)$ - $\log(q)$  plane— sufficiently restricted as to define a theoretical fundamental plane with scatter less than that of the  $M - \sigma_o$  relation. A least-square fit to this distribution of points gives  $\kappa = 0.98$  in eq. 10. Thus, by eqs. 12, this yields a dynamical fundamental plane relation near that implied by the Newtonian virial theorem for homologous systems, i.e.,  $\alpha = 2, \gamma = 1$ . Because the scatter in  $q$  about this power law relation is much less than the total range in  $q$ , the scatter perpendicular to the dynamical fundamental plane is very much reduced.

For comparison with observations, it must be realized that both the  $M - \sigma$  and fundamental plane relations are altered by the way in which elliptical galaxies are actually observed. Specifically, it is not the velocity dispersion along the very central line-of-sight,  $\sigma_o$ , which is measured, but rather the l.o.s. velocity dispersion,  $\sigma_d$ , within some finite-size aperture with radius  $r_d$ . The data of Jørgensen et al. have the advantage that all measured velocity dispersions are corrected to a circular aperture with a fixed linear diameter of 1.6 kpc for  $H_o = 75$ . The appearance of a fixed linear scale has the effect of introducing an additional dimensionless parameter into dynamical relationship, eq. 9; i.e.,  $q$  also becomes a function of  $r_d/R_\sigma$  where  $R_\sigma = \sigma_d^2/a_o$  is the MOND length scale appropriate to the system. However, this parameter can be absorbed if it is expressed as  $r_d/R_\sigma = \sigma_m^2/\sigma_d^2$  where  $\sigma_m = \sqrt{a_o r_d} = 54.4$  km/s.

When we observe the polytropic models in the same way as real galaxies (determining the volume emissivity-weighted l.o.s. velocity dispersion in the inner projected 0.8 kpc), the distributions of velocity dispersions and effective radii may be compared directly to the observations of Jørgensen et al. This is done in Fig. 4 where we see that such models can account for the observed range in these properties provided



**Figure 5.** The top panel is the mass-velocity dispersion relationship ( $M - \sigma_d$ ) for the ensemble of anisotropic polytropes. The polytropic index ranges between 12 and 16 and the scaled anisotropy radius is greater than 0.2. This is actually a collection of  $M - \sigma_d$  relationships, one for each combination of the two model parameters. The bottom panel shows the result of entering a third parameter, i.e., the best-fit fundamental plane relation. Here  $\log(\sigma_d) + \gamma' \log(r_e)$  is plotted against  $\log(M)$  and  $\gamma'$  is chosen to give the lowest scatter. The resulting slope is about 1.76 with  $\gamma' = 0.56$ , as implied by the  $q' - \Sigma$  relation shown in Fig. 3.

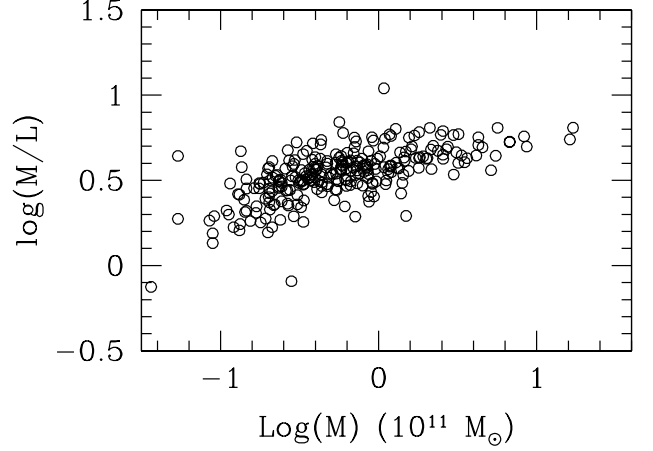
that free parameters cover their allowed ranges:  $12 \leq n \leq 16$  and  $0.2 \leq \eta$ . That is to say, the set of models must be non-homologous in order to explain the dispersion in observed properties.

For these realistically “observed” models, the dependence of  $q$  on  $\sigma_d$  is also found to be power law; thus, we may rewrite eq. 9 as

$$\sigma_d^4 = q'(\Sigma/\Sigma_m)[\sigma_d/\sigma_m]^\lambda G M a_o; \quad (13)$$

i.e., I explicitly write the dependence of  $q$  on  $\sigma_d$  leaving the quantity  $q'$  as a pure function of surface density. In Fig. 3 we see that the dependence of  $q'$  on surface brightness is well-represented by a power law with roughly the same exponent as the  $q$  dependence. Thus the  $M - \sigma$  relation becomes  $M \propto \sigma_d^{(4-\lambda)}$  and the fundamental plane exponent in eq. 12a is  $\alpha = (4 - \lambda)/(\kappa + 1)$ . From the models it is found via least-square fits that  $\lambda = 0.53$  and  $\kappa = 0.98$  implying

$$M/(10^{11} M_\odot) = 2 \times 10^{-8} \sigma_d(\text{km/s})^{3.47} \quad (14a)$$



**Figure 6.** The mass-to-light ratio of galaxies in the samples of Jørgensen et al. as a function of mass where mass is estimated from the fundamental plane relation shown in Fig. 5 (for the anisotropic MOND polytropes). The mean  $M/L$  is 3.6 with a 30% scatter. As in the strictly Newtonian case there is a weak dependence of  $M/L$  on  $M$ .

for the mass-velocity dispersion relation and

$$M/(10^{11} M_\odot) = 3 \times 10^{-5} [\sigma_d(\text{km/s})]^{1.76} [r_e(\text{kpc})]^{0.98}. \quad (14b)$$

These  $M - \sigma$  and dynamical fundamental plane relations are shown for the 360 models in Fig. 5. The scatter about the dynamical fundamental plane is a factor of 10 smaller than that about the  $M - \sigma$  relation.

Note in eqs. 14 that for this restricted set of models there is a definite scaling of both the  $M - \sigma$  and the dynamical fundamental plane. Using eq. 14b to calculate the mass of the galaxies in the Jørgensen et al. sample, one finds the distribution of  $M/L$  shown in Fig. 6 which is a log-log plot of  $M/L$  against the calculated mass. Here it is found that  $\langle M/L \rangle = 3.6 \pm 1.2$  in solar units ( $H_o \approx 75$ ) and  $M/L \propto M^{0.2}$ . This, of course, ignores possibly important effects such as deviations from spherical symmetry and systematic rotation, and the fact that real galaxies are almost certainly not characterized by a pure polytropic velocity dispersion-density relation. Bearing these potential dangers in mind, one could also extrapolate eq. 14b down to globular clusters. For a sample of globular clusters tabulated by Trager, Djorgovski & King (1993) and by Pryor & Meylen (1993), I find that  $\langle M/L \rangle = 1.7 \pm 0.8$  in solar units.

Thus, MOND polytropic spheres in the range  $n=12$  to  $n=16$  with radial anisotropy beyond the effective radius not only reproduce the observed distribution of galaxies in the  $r_e - \sigma_d$  plane but also provide a reasonable value for the mass-to-light ratio of ellipticals and a weak dependence of  $M/L$  on mass. As is evident from eq. 9, the usual MOND  $M \propto \sigma_o^4$  relation remains, albeit with large scatter due to the necessary deviations from homology. Considering the manner in which the central velocity dispersion is actually measured (within a fixed circular diaphragm) the relation is altered to  $M \propto \sigma_d^{3.47}$ . Further, considering the necessary dependence of  $M/L$  on  $M$ , the predicted Faber-Jackson relation becomes  $L \propto \sigma_d^{2.78}$  which is consistent with the data of Jørgensen et al.; i.e., a least square fit to the  $\log(L) - \log(\sigma_d)$



distribution for this sample of early-type galaxies yields a slope of  $2.6 \pm 0.8$ .

The predicted dynamical fundamental plane eq. 14b can be converted into the more commonly used form by making use of the relation  $M = 2\pi\Sigma r_e^2$ ; then one finds  $r_e \propto \sigma_d^{1.73} \Sigma^{0.98}$  where  $\Sigma$  is the mean *mass surface density* within  $r_e$ . With  $M/L \propto M^{0.17}$  we would then predict a fundamental plane of

$$r_e \propto \sigma_d^{1.23} I_e^{-0.84} \quad (15)$$

where  $I_e$  is the mean surface brightness within  $r_e$ . Within the uncertainties this is identical to the fundamental plane defined by the observations of Jørgensen et al. (1995a,b).

Given the approximations involved in these calculations (primarily the polytropic gas assumption and the specific radial dependence of the anisotropy parameter), the actual exponents of the  $M - \sigma$  and fundamental plane relationships are less important than the fact that MOND predicts a fundamental plane relation with a scatter which is a factor of 10 less than that about the  $M - \sigma$  relation. This is true in spite of the fact that the set of models must be non-homologous in order to explain the range of observed properties— surface density and effective radius. This arises as a natural aspect of the basic dynamics of systems which extend into the regime of modified dynamics and need not be accounted for by complicated conspiracies in the process of galaxy formation. Moreover various mechanisms for the subsequent dynamical evolution of ellipticals (e.g. merging, cannibalism) would not effect this relationship. All that is required is that stellar velocity field in ellipticals not deviate too dramatically from being isothermal and possess a radial orbit anisotropy similar to that described by eq. 4.

## 5 CONCLUSIONS

The essential results of these calculations can be summarized as follows:

1. The dynamics of high surface brightness elliptical galaxies span the range from Newtonian within the effective radius to MOND beyond. The mean surface density within  $r_e$  is at least twice as large as the MOND surface density, and the internal accelerations are too large to be within the domain of modified dynamics. This is consistent with the fact that there is no large mass discrepancy or, viewed in terms of dark matter, no need for dark matter within the bright inner regions. However, MOND isothermal spheres have a mean surface density which is roughly one-tenth the critical surface density within  $r_e$ ; they are almost entirely pure MOND objects. This effectively rules out these objects as models for elliptical galaxies.
2. In order to reproduce the observed global properties of high surface brightness elliptical galaxies in the context of MOND, it is necessary to introduce deviations from a constant velocity dispersion and strict isotropy of the velocity field in the outer regions. This may be done, in an approximate way, by considering MOND polytropes in the range  $n=12$  to  $n=16$  with a radial orbit anisotropy beyond an effective radius ( $r_a > 0.75r_e$ ). These objects exhibit the mean

decline of line-of-sight velocity dispersion with projected radius observed in elliptical galaxies. Moreover, such models provide reasonable representations of elliptical galaxies with respect to the distribution by velocity dispersion and effective radius (Fig. 4). In order to match these observations, the models must cover a range in the parameter space of polytropic index and scaled anisotropy radius which implies that strict homology is broken. This breaking of homology leads to considerable scatter in the mass-velocity dispersion relation (and the implied Faber-Jackson relation) while introducing a third parameter which is the mean surface density or effective radius. The intrinsic scatter about this dynamical fundamental plane is much lower than that about the mass-velocity dispersion relationship because of the relative insensitivity of this theoretical relationship to deviations from homology (Fig. 5). Both the theoretical  $M - \sigma$  and fundamental plane relationships are modified when one considers that the central velocity dispersion is actually measured within a finite size aperture corrected, in the Jørgensen et al. observations, to a fixed diameter of 1.6 kpc for all galaxies in their samples.

3. These calculations are highly idealized and apply strictly only to spherical systems with a perfect polytropic equation of state. Moreover, the fact that the models cover a range of internal accelerations around  $a_o$  means that the detailed structure is dependent upon the assumed form of the MOND interpolation function,  $\mu(x)$  (eq. 2). Nonetheless, when the derived dynamical fundamental plane relation is used to estimate the mass of elliptical galaxies from the observed central velocity dispersion and effective radius, one finds, for the galaxies in the samples of Jørgensen et al., a mean mass-to-light ratio of  $3.6 M_\odot/L_\odot$  with a dispersion of 30% and a weak dependence of this M/L on galaxy mass (as in the strictly Newtonian case). Such a M/L would seem quite reasonable for the older stellar populations of elliptical galaxies. When the predicted dynamical fundamental plane relation is converted into an observed relationship (on the  $r_e$ ,  $\sigma_d$ , and  $I_e$  parameter space), the Jørgensen et al. result is recovered if  $M/L \propto M^{0.17}$ .

The principal conclusion is that the existence of a fundamental plane with lower intrinsic scatter than that of the Faber-Jackson relation is implied by modified dynamics, given that high surface brightness elliptical galaxies cannot be represented by pure MOND isothermal spheres. It may be argued that Newtonian dynamics also predicts a fundamental plane via the traditional virial theorem, and therefore the existence of such a relationship in no sense requires modified dynamics. It is true that the fundamental plane by itself would not be a sufficient justification for modified dynamics. However, a curious aspect of the Newtonian basis for the fundamental plane is the small scatter about the observed relation in view of the likely deviations from homology in actual elliptical galaxies. The advantage of MOND in this respect is the existence of a single dynamical relationship (eq. 12 or eq. 13) for a range of non-homologous models. Because an additional dimensional constant,  $a_o$ , enters into the structure equation (eq. 2), MOND self-gravitating objects are more constrained than their Newtonian counterparts.

In this respect, it is worthwhile to emphasize that a pure Newtonian self-gravitating object with a central line-of-sight velocity dispersion of 200 km/s can have any mass. But an object with this same velocity dispersion and which extends at least partially into the regime of modified dynamics can only have a galaxy-scale mass. It is the proximity to being isothermal which requires that elliptical galaxies extend into the regime of modified dynamics. If this one basic requirement is satisfied, MOND inevitably imposes boundary conditions on the inner Newtonian solution— boundary conditions which restrict these objects to lie on such a well-defined plane in the three dimensional space of observed quantities in spite of detailed variations in the structure between individual objects. Structural variety does, however, lead to a large intrinsic scatter in the  $M - \sigma$  relation because each distinct class of objects characterized by an appropriately scaled radial dependence of velocity dispersion and degree of anisotropy exhibits its own  $M - \sigma$  relation with a different normalization. None-the-less, a Faber-Jackson relation does exist and remains as an imprint of modified dynamics on nearly isothermal hot stellar systems.

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